Examining the Impact of an Integrative Method of Using Technology on Students' Achievement and Efficiency of Computer Usage and on Pedagogical Procedure in Geometry

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Received: 13 October 2011 Revised: 14 July 2012

In the current study we follow the development of the pedagogical procedure for the course "Constructions in Geometry" that resulted from using dynamic geometry software (DGS), where the computer became an integral part of the educational process.Furthermore, we examine the influence of integrating DGS into the course on students' achievement and efficiency of computer usage.

The research population consisted of second-year students in an undergraduate education program for junior high school mathematics teachers, and is characterized by almost no experience with construction problems. The results of our previous studies pointed to a gap between our intention to integrate DGS and its actual usage in the classroom (by both the students and the instructor). The results of the current study show that the implementation of the newly developed teaching procedure improved the students' usage of the computerized environment as well as their achievement for both simple and difficult types of problems.

1 INTEGRATING DYNAMIC GEOMETRY SOFTWARE IN GEOMETRY CLASSROOMS

In the current study we concentrated on the modification of the pedagogical procedure of the course "Constructions in Geometry" while integrating dynamic geometry software (DGS) in the classroom.In particular, we were interested to test whether such integration improves students' learning. The described course is given for sophomores in the teacher education program for in-service and pre-service secondary school teachers in the Mathematics Teaching Department at Achva College of Education. The course provides the students with important geometrical knowledge, and is critical for proper understanding of several advanced courses in the education program. At the same time, our research population can be characterized by a relatively low level of geometric thinking and practically no experience with construction problems. Therefore we suggest that the implementation of new approaches based on computer technologies can contribute to their development. The decision to use such approaches seems to us almost obvious in light of numerous recent studies indicating that integration of technology into mathematics classrooms has a significant impact on the teaching of mathematics by supporting and enhancing the acquisition of mathematical learning (Monaghan, 2001; Hollerbands, 2007).

Furthermore, our own experience (Barabash, Gurevich and Yanovsky, 2009) and many other recent studies (Abboud-Blanchard and Lagrange, 2006; Christou, Mousoulides, Pittalis and Pitta-Pantazi, 2005; Goldenberg and Cuoco, 1998; Healy and Hoyles, 2001; Kasten and Sinclair, 2009; Oner, 2009; Sinclair, 2004) indicate that the usage of a dynamic geometry environment in geometry classrooms permits learners to turn studying geometry in general, and constructions in geometry in particular, into a process of experimentation and exploration as well as justification and verification, while the dynamic features of DGS enable users to implement the widely discussed maxim that knowing mathematics means doing mathematics. DGS provides students with the opportunity to experiment with different geometrical objects and to receive immediate feedback. DGS also enables students to validate their solutions by merely dragging the resulting figure over to the required one to make sure that they correspond. The ability to drag figures helps students to check their conjectures, which may lead to a solution; that is, it allows students to confirm or to refute a conjecture and then, if necessary, to move on to another conjecture. The computer environment gives students a wide range of appropriate approaches and possible ways to accomplish a task and at the same time does not allow them to act against geometrical laws (Barabash, Gurevich and Yanovsky, 2009).

Unfortunately, in spite of all these benefits, such integration meets serious difficulties in practice on the part of teachers as well as students (Cuban, Kirkpatrick and Peck, 2001). It is commonly admitted that there is a certain discrepancy between the effective models of instruction based on the integration of new technology into teaching and its actual role in classrooms (Abboud-Blanchard and Lagrange, 2006). Our previous results also pointed to a gap between our intention to integrate DGS and its practical usage during lessons (by both the students and the instructor). Namely, we observed that the students had serious difficulties in dealing with the investigation of solutions using DGS (Barabash, Gurevich and Yanovsky, 2009).

As has been reported previously, the way to effectively integrate technology into the mathematics classroom depends on the teacher, on his experience of using the technology, and on his proper selection of mathematical tasks that permit him to take advantage of the technology's features (Mously, Lambdin and Koc, 2003; Lawless and Pellgrino, 2007). It is generally known that teachers usually resist technological innovations, claiming that they impede teaching by taking time and distracting students from the subject (Wenglinsky, 1998). Therefore, using computers and learning how to work with particular software becomes a real challenge for teachers. Many teachers don't feel comfortable with the new technology. Even after they have gotten acquainted with the chosen technology, they may still be a long way from being able to effectively integrate it into their teaching.

Koehler and Mishra (2008) refer to the design of instruction during the integration of technology. They claim that the educator should look for effective ways of integrating technology, pedagogy and content knowledge.

Numerous studies that focused on integrating technology in mathematics education indicate that various tools each have specific capacities, although each of them also has its constraints, and technologies usually are not unbiased. The instructor must be able to choose the most appropriate tool for each specific assignment. The main questions are how to take advantage of each tool in the most effective and creative way (for example, Calder, 2011; Lagrange, Argitue, Laborde and Trouche, 2003; Pitard, 2011) as well as how to integrate it in the classroom (Harris and Hofer, 2009; Kosma, 2003; Miodusar, Nachmias, Tubin and Forkosh-Baruch, 2003).

Referring to students' attitudes towards technology integration in math teaching, several studies (Nuggent, Soh, and Samal, 2006) have suggested that technology could motivate students to learn mathematics. At the same time, numerous studies have indicated that not all students are confident in the use of technology, nor are they all convinced of the benefits of computer-aided teaching (Trouche, 2005). D'Souza and Wood (2004) found that students frequently mistrusted software and felt more comfortable with traditional methods; namely, they preferred using pen and paper, because this was more reliable and easier.

2 TECHNOLOGY BECOMES AN INTEGRAL PART OF CLASS PEDAGOGY

Deaney and Hennessy (2011) discuss the importance of looking for an adaptive approach to harnessing technology that can address a wide diversity of individual differences encountered in a very mixed class of students. Salomon (1997) points to another important aspect of the integration of technology in education. He suggests that the mode of integration should be congruent to the pedagogical rationale. Recent studies support this approach. It has been found that technology usage offers a range of pedagogical opportunities and may support students in directing their own learning and exploring mathematics, equipping them to share their findings with the teacher and the class with more confidence (Pierce, Stacy and Wander, 2010; Pierce and Stacy, 2010).

Unkefer, Shinde and McMaster (2009) propose that the implementation of technology in the educational process induces encouraging teachers to look for the appropriate learning environment and pedagogical procedure. According to this conception, the principle of integrating a dynamic environment into the educational process entails continual modification of the classroom and the teaching methodology. Furthermore, the introduction of new technology ought to influence the curriculum and the physical arrangement of the classroom. The above concept is based on Hooper and Reiber's (1995) proposal, where they presented their five-phase model for adapting educational technology in the classroom, as follows:

1. Familiarization: The instructor's initial exposure to and experience with the technology.
2. Utilization: Attempts by the instructor to use the technology in the classroom without any commitment or intention.
3. Integration: The technology becomes an integral part of the course.
4. Reorientation: The reconsideration and the reconceptualization of the purpose and functions of the technology in the classroom.
5. Evolution: The continual modification of the classroom according to the obtained experience of the integration of technology in the classroom.

The subject of the current research is an evaluation of the pedagogical procedure that we have newly developed.We assume that it is relevant to describe very briefly the history of the ways in which we have integrated computerized tools in teaching construction problems in Geometryduring the last approximately 10 years. Here we refer to three critical stages of this process.

1. At first (in the 2001 academic year) we taught the course **"Computer Usage in Math Teaching"**. During this course students got acquainted with Dynamic Geometry Software (DGS) and practiced basic construction problems that they had learned in the course **"Plane Geometry"**.

We tested the impact of computerized tools on geometry teaching (Gurevich, Gorev and Barabash, 2005). The results obtained showed that the students found it difficult to decide when and for what reasons to turn to the computer. At the same time the results indicated the necessity of using computers in mathematics lessons in a more intensive manner.

2. Two years later (in the 2003 academic year), we implemented the same format for another experiment with a new group of students that took in parallel two courses: "**Constructions in Plane Geometry**" and"**Computer Usage in Math Teaching**". The subject of the first course was the solution of construction problems in geometry. We used the generally accepted geometry approach according to which the solution of construction problems includes four essential steps: analysis, construction, proof, and investigation (Yaglom, 1962). A straightforward linkage was established between the two courses, so that in "**Computer Usage in Math Teaching**" the students solved with the DGS the same problems that they solved analytically in "**Constructions in Plane Geometry**". We developed a teaching method which

provided the students with a set of guidelines for using the DGS throughout the four stages of solution (Barabash, Gurevich and Yanovsky, 2009).

Our study published in 2009 was conducted in order to investigate the impact of the above method, and it showed that the computer usage was quite effective, especially for the group of low-achievers in solving more complicated problems. Furthermore, it was found that the students used the DGS intensively at the analysis and construction stages. However, it was observed that the students had serious difficulties when dealing with the proof and investigation stages of solution.

3. Based on those results, we decided to merge those two courses into one, taught (since the 2008 academic year) in a computerized classroom, so that the computer became an integral part of the educational process.

The main question that we posed in the current research was whether the newly developed teaching method contributes to students' achievement and the efficiency of their computer usage. Moreover, we were interested to learn about the impact of an integrative method of using technology on pedagogical procedure.

3 METHODS

Research population

The research population was two groups of second-year students in the educational program for secondary-school mathematics teachers of the Math Teaching Department atAchva College of Education. The groups were drawn from two different years: Group 1 (18 students) from 2003, and Group 2 (13 students) from 2008, as was presented in the previous section.

Research format

The students in Group 1 took the two courses "**Constructions in Plane Geometry**" and "**Computer usage in Math Teaching**". Both courses were taught simultaneously during one academic year by two different teachers so that in "**Computer usage in Math Teaching**" the students solved with DGS the same problems that they solved analytically in "**Constructions in Plane Geometr**y". The students in Group 2 studied all subjects covered in the above two courses in one merged course,which was taughtin a computerized classroom**.**

In order to distinguish between the two teaching modes described above, we call the first (2003, Group 1) the Computer-Exercising procedure of teaching (CET), and the second (2008, Group 2) the Computer-Integrated procedure of teaching (CIT).The groups were chosen as being representative of the CET and CIT pedagogical procedures, respectively.

Since the aim of the current research was to test the impact of the integrated method of teaching, in both groups,at the end of each academic year, the students were tested using the same problem set, which consisted of four construction problems: two that were relatively simple and two that were more complicated. The problem setwas supplemented with a questionnaire (see Appendix 1) regarding the attributes of the contribution of the computer application at each of the four stages of solution mentioned above. It should be mentioned that we used the attributes specified in our previous study, "Usage of computerized environment in the course 'Plane Transformations and Constructions in Geometry'" (Barabash, Gurevich and Yanovsky, 2009).

The students were asked to choose two problems (see Appendix 2): one of a simple type and one of a more difficult type, and then to answer the questions. To meet our requirements the students had to present their solutions both in writing and on the computer. Here it is important to emphasize that we believe the comparison between the two groups is legitimate since in both groups the problem set was the same, and in addition our requirements concerning both analytic and computerized solution methods were the same. Moreover, the students in both groups had more or less the same math abilities (since in both groups the students had practically no previous experience with construction problems) as well as the same level of computer proficiency (since in the both groups the students had been practicing the DGS for one semester).

Data processing

The data were obtained both from the students' answers to the problems and the questionnaire.

The students' answers to each problem were evaluated with respect to the followingtwomeasures:

- Application of the computerwas defined as at least one optionmarked in the questionnaire in reference to each of the 4 stages of solution (see Appendix 1).
- Successful solution was defined as a score of more than 7 for students' answers at each stage for each problem (4 grades in all, each on a scale of 0 to 10).

The study's independent variables are:

- The teaching procedure: CET and CIT, implemented in Group1 and Group 2, respectively;
- The problem's difficulty level (a simpler type and a more difficult type);
- The four stages of solution (Analysis, Construction, Proof, Investigation).

The dependent variables (each measured at each stage of solution) are:

- Computer contribution (the percentage of those students who referred to the computer application among those who succeeded in solving the problem);
- Computer efficiency (the percentage of students who succeeded in solving the problem, among those who referred to the computer application);
- Students' success(the percentage of successful solutions per stage of problem).

To analyse the data, χ^2 tests were conducted to compare each of the three dependent variables along each of the three independent variables (as described above).

In the qualitative analysis of the teaching procedure that we developed, in addition to the open questions of the questionnaire (Questions 2 and 3 in Appendix1) we also used our researchers' journal dealing with planning the lessons. The content analysis was performedaccording to Denzin and Lincoln (2005).

4 OBSERVATIONS AND RESULTS

The impact of anintegrative method of using technology on the pedagogy of the course "Constructions in Geometry"

The current teaching procedure arose from the results and observations of our previous studies that dealt with the integration of technology in Geometry instruction, as was described above. The course in question, "Constructions in Plane Geometry", was taught in a computerized classroom,which meant that each lesson was planned by taking into account both the dynamic features of the DGS and our experience of its usage.

We notedin our researchers' journal that each new topic was explained and presented both analytically and using the computer, so that the students had the opportunity to explore the topic themselves by means of the DGS. In particular, each exercise was solved both analytically and by using the DGS at all four stages of solution: analysis, construction, proof and investigation. Our aim was to teach the students to use the DGS tool at each of these stages. A special emphasis was placed on DGS usage at the proof and investigation stages, since the previous results pointed to inefficient usage of the computer by the students at those stages.

Referring to the changes that occurred in the pedagogy of the course, we found it to be very important to test its impact on computer contribution to the students' solution of the problems; on the efficiency of their computer usage and on the students' achievement.

Computer contribution

Aiming to analyse the contribution of the DGS, we examine only those students who succeeded in solving the given problems.In Figure 1, we present the rate of computer contribution (the percentage of those students who referred to the computer application among those who succeeded in solving the problem):

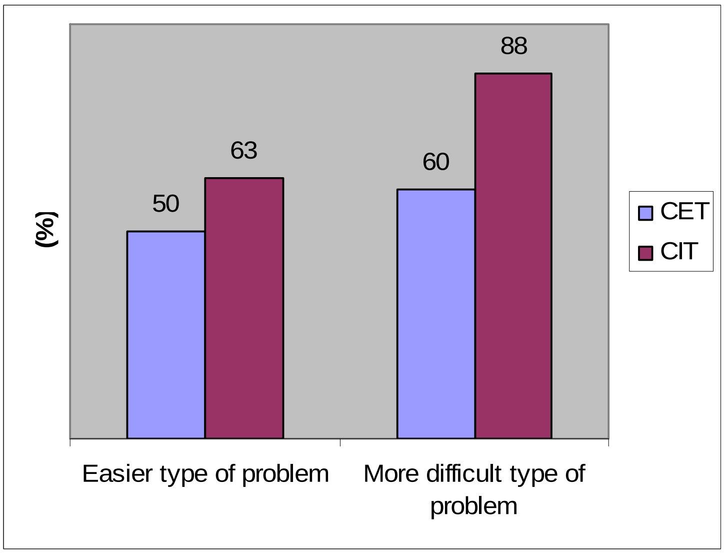

Figure 1 The rate of computer contribution in two groups by type of problem.

The results are presented in Figure 1. A χ^2 test was performed to compare the above percentages within each ofthe two types of problems across groups (Easier type: χ^2 = 0.336, p = 0.562; more difficult type: χ^2 = 1.675, p = 0.196). In addition, a χ^2 test was performed to compare the percentages within each group across types of problem (Group1: χ^2 = 0.248, p = 0.619, Group 2: χ^2 = 1.33, p = 0.248). The above results were a trigger for more detailed tests.

We checked the contribution of the DGS at each of the four stages of solution for each group and both kinds of problem. In Table 1 we present the results for the rate of computer contribution at different stages of solution. χ2 testswere performed to compare the results by type of problem as well as by group.

The results in Table 1 show that there is no significant difference in rate of computer contribution with respect to the difficulty of the problem in both groups. At the same time, the results indicate a significantly higher computer contribution in Group 2 at the Investigation stage.

	Analysis		Construction		Proof		Investigation	
	CET	CIT	CET	CIT	CET	CIT	CET	CIT
Easier type of problem	92%	50%	64%	70%	33%	45%	50%	100%
More difficult type of problem	87%	85%	83%	78%	0%	45%	0%	80%
χ^2 test by problem type	χ^2= 0.13, p=0.72	χ^2= 3.20 p= 0.07	χ^2 = 0.73, p = 0.39	χ^2=0.15, p=0.70	χ^2 = 0, p = 1	χ^2 = 0, p = 1	χ^2= 1.33, p=0.25	χ^2=1.32, p= 0.25
χ^2 test by group	χ^2 = 2.95, p = 0.09		χ^2 = 0.06, p =0 .80		χ^2 = 0.47, p = 0.49		χ^2 = 6.52, p = 0.01	

(The significant results ($p \leq 0.05$) are marked by highlighting.)

Table 1 Computer contribution in two groups for two types of problem at each stage of solution.

Computer efficiency

Moreover, we were interested in testing the effectiveness of the DGS at the different stages of solution and in comparing the results within the two Groups for the two types of problems.

In Table 2 we present the percentages of students who succeeded in solvingeach stage of the problems, among those who pointed out the computer application at each stage.

The results point to significantly lower computer effectiveness in more difficult type of problem than in the less difficult one in Group 1 at two stages, namely the Construction stage and the Proof stage.

Furthermore, as can be seen from Table 2, the effectiveness of the computer usage is significantly higher in Group 2 than in Group 1 at the Construction stage, at the Proof stage and at the Investigation stage of the solution for both types of problems.

	Analysis		Construction		Proof		Investigation	
	CET	CIT	CET	CIT	CET	CIT	CET	CIT
Easier type of problem	75%	71%	82%	78%	50%	83%	20%	67%
More difficult type of problem	44%	83%	33%	87%	0%	100%	20%	63%
χ^2 test by problem type	χ^2=3.24, p= 0.07	χ^2= 0.26, p = 0.61	χ^2 = 6.0, p= 0.01	χ^2=0.27, p = 0.6	χ^2= 5.88, p = 0.01	χ^2=3.44, p=0.06	χ^2 = 0, p = 1	χ^2= 0.03, p= 0.86
χ^2 test by group	χ^2 = 1.24, p = 0.26		χ2 = 3.68, p = 0.05		χ2 = 6.6, p = 0. 01		χ2 = 7.62, p = 0.01	

(The significant results ($p \leq 0.05$) are marked by highlighting)

Table 2: Computer effectiveness in two groups for two types of problem at each stage of solution.

Students' success

We also analyzed the rate of success (the percentage of successful solutions) in both groups for the two problems at each stage of the solution. These results are presented in Table 3.

The results indicate a significantly lower rate of success for the more difficult type of problem than for the less difficult one in Group 1 at two stages, namely the Construction stage and the Proof stage.

At the same time the rate of success is significantly higher in Group 2 (2008) at the Proof stage and at the Investigation stage.

	Analysis		Construction		Proof		Investigation	
	CET	CIT	CET	CIT	CET	CIT	CET	CIT
Easier type of problem	72%	77%	78%	77%	67%	85%	11%	46%
More difficult type of problem	44%	85%	33%	69%	0%	85%	11%	38%
χ^2 test by problem type	χ^2=2.86, p=0.09	χ^2=0.25, p= 0.62	χ^2= 7.2, p= 0.01	χ^2=0.19, p= 0.66	χ^2 = 18, p= 0.00	χ^2 = 0, p = 1	χ^2 = 0, p = 1	χ^2=0.05, p = 0.82
χ^2 test by group	χ^2 = 0, p = 1		χ^2 = 1.99, p = 0.16		χ^2 =16.03,p = 0.00		χ^2 = 8.01,p = 0.00	

(The significant results ($p \leq 0.05$) are marked by highlighting)

Table 3 Students' success in two groups for two types of problem at each stage of solution.

5 DISCUSSION

In the current study we analyse the changes we made in the pedagogical procedure. We observed that using DGS tools in teaching leads toward a revision of routine pedagogical procedures. Evaluating our experience of using the DGS in teaching for more than 10 years, we suggest that it is in good agreement with the five-phase model for adapting educational technology in the classroom (Hooper and Reiber, 1995). At the beginning of this period (the 2001 academic year) the adaptation of educational technology in the classroom consisted of familiarization and utilization (according to Hooper and Reiber, 1995), which meant occasional usage of technology for selected topics in the course. Then, at a later stage in the course's evolution (the 2003 academic year), our pedagogical procedure corresponded to a stage of integration (ibid.), where DGS became an integral part of the course. At the current stage (since the 2008 academic year) the teaching procedure can be characterized as reorientation and evolution (ibid.), in which the lesson plan is changed due to the obtained experience of integrating DGS in the classroom. The students are encouraged to conjecture and explore each presented topic, theorem or problem.

We found it important to examinethe impact of these changes in pedagogical procedure on our students' acquisition of mathematical learning;therefore,we compared students'mode of computer usage and their achievements under the two pedagogical procedures, namely CET and CIT.

The analysis of computer contribution does not reveal any difference between CET and CIT methods.At the same time the more detailed analysis of computer contribution at the four stages of solution forthe two types of problems reveals no significant difference with respect to problem type, but a significant difference was detected between the two methods atthe Investigation stage of solution. It was observed that the implementation of the computer-integrated teaching procedure significantly increased computer contribution at the Investigation stage (the most problematic stage of solution for the students). We consider this result quite important since our previous results from the experiment conducted in 2003 (Barabashet al., 2009) pointed to intensive usage of the DGS only at the Analysis and Construction stages. Furthermore, we suggested that a relatively low rate of computer contribution at the Proof stage seems to be rather natural, while a low rate at the Investigation stage might be referred to the students' lack of practice and experience, which results in a lack of awareness of the possibilities of computer usage for such purposes. The newly implemented pedagogical procedure in the present study consisted of very detailed instructions on how to use the DGS at each specific stage of solution, accompanied bydemonstration and practice of the appropriate examples. We assume that these changes underlie the observed improvement.

In addition to the analysis of computer contribution, we tested the effectiveness of the DGS at different stages of solution. We found that at three of the four stages of solution, namely, the Construction, Proof and Investigation stages, the effectiveness was significantly higher under the CIT method. This means that the implementation of the computer-integrated teaching method significantly increased the effectiveness of computer usage during most stages of solution for these students.

Moreover, it seems important to emphasize here that, by contrast to the results from 2003 (for the CET method), which pointed to lower effectiveness of the DGS usage by the students in solving the more difficult problem at the Construction stage and the Proof stage, the results from 2008 (for the CIT method) showed an increase in effectiveness even at these two stages in the more difficult problem.

Finally, we compared the students' achievements in the two groups by computing students' success at all stages of solution for the two problems types. The results indicated significantly higher success rates at two stages, namely the Proof and Investigation stages,under the CIT method.

In summary, the obtained results show that implementation of the Computer-Integrated procedure of teaching (CIT) improves students' usage of the computerized environment for both simple and difficult types of problems at most stages of solution. Moreover, the implementation of CIT enables students to improve their achievement significantly even for relatively difficult problems.

Based on these results, we suggest that the Computer-Integrated procedure of teaching (CIT) enables students to construct and assimilate more complete and wide-scale knowledge in comparison to the Computer-Exercising procedure of teaching (CET). We suggest that the findings of the current research are in good agreement with the recent results that indicated that proper integration of technology into mathematics classrooms significantly improvesthe acquisition of mathematical learning (Hollerbands, 2007; Kasten and Sinclair, 2009; Oner, 2009).Furthermore, based on the obtained results we assume that the CIT mode of teaching might be especially effective for students when dealing with difficult aspects of learning material. This assumption fits the results of Kasten and Sinclair (2009)regarding the tendency by teachers to choose activities that solve their own current and persistent difficultiesin delivering learning material. In addition, we believe that the described pedagogical procedure can provide an appropriate learning environment where learners with different profiles or styles can work together productively, as was noted by Deaney and Hennessy (2011).

This study deals with relatively small groups of students; but, we believe that our results might point to a certain tendency in teaching and learning when technology is introduced in a classroom. Although the current study refers only to two groups of students taken from two different academic years, the development of the course havetaken place over a decade, and the two chosen groups are typical of experiences with CET and CIT,respectively.

We hope that the procedure of teaching we developed can provide future teachers with effective models of instruction.

Special thanks go to Dr. Marita Barabash for her fruitful comments.

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BIOGRAPHICAL NOTES

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APPENDIX 1. THE QUESTIONNAIRE PRESENTED TO THE STUDENTS.

1. Present the detailed solution of the problem.
2. Describe how you applied the computer: e.g., you did not try to solve a problem before you were able to use the computer, and then you used it alternately with paper and pen; or: you did not need it at all; or: you used it to draw a sketch and then solved the problem without the computer application; etc.
3. How do you assess the contribution of the computer to your solution?

Please fill in the following table (please mark the appropriate mode of computer usage):

Computer application mode	Problem No.1	Problem No.2
At the Analysis stage: As a tool to understand the problem As a tool in the search for a solution As a tool for auxiliary constructions Other (please indicate in detail)		
At the Construction stage: As a tool to test the correctness of the construction As a tool in the search for a construction strategy As a tool in the search for other solutions Other (please indicate in detail)		
At the Proof stage: How did the usage of the computerized tools contribute to the proof process? Did the computerized tools help you in checking the correctness of the proof? Other (please indicate in detail)		
At the Investigation stage: As a tool to analyze the conditions for existence and uniqueness of the solution As a tool for testing possible solutions Other (please indicate in detail)		

Appendix 2. The problems presented to the students.

Problem 1 (the simpler type):

Construct a triangle given two of its sides and the altitude to the third side (b, c, h_a).

Problem 2 (the simpler type):

Construct a triangle given one of its sides, the altitude to this side and the radius of its circumcircle (a, h_a, R).

Problem 3 (the more difficult type):

Construct a quadrangle given three of its sides and the two angles adjacent to its fourth side.

Problem 4 (the more difficult type):

Construct a triangle given two of its sides and the difference of the angles adjacent to its third side.